Damping optimization and bounds on eigenspaces

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We consider a linear vibrational system

$$M\ddot{x} + D\dot{x} + Kx = 0, \qquad x(0) = x_0, \qquad \dot{x}(0) = \dot{x}_0,$$

where, matrices M and K represent mass and stiffness, respectively, which are either positive definite or semidefinite of order n. The damping matrix is $D = C_u + C$, where C_u represents internal damping and C is an external damping which is usually low rank and semidefinite.
The problem of damping optimization of systems like this is very demanding and can be approached form a several different aspects. Thus, if A a matrix obtained using appropriate linearization of the corresponding QEP $(\lambda^2 M + \lambda(C_u + C) + K)x = 0$, as the one approach we can considers a behaviour of the spectrum of the matrix A. For example one can use *spectral abscissa criterion*, that is $\min_i \mathrm{Re}(\lambda_i)$, where λ_i are eigenvalues of the matrix A. Another approach considers the solution X of the corresponding Lyapunov equation $AX + XA^T = -I$. Within this approach one can minimize the trace of X or the norm of X as the function of the external damping C.
Since, this kind of optimization processes are computationally very demanding it is useful to approximate original systems with smaller one to accelerate optimization processes. This can be obtained using a dimension reduction, for example, one can use reduction using the r columns of the matrix Φ which simultaneously diagonalize pair (M, K).
The quality of obtained approximations strongly depend on chosen subspaces, thus one of our aims is to show how one can bound the norm of sines of canonical angles between subspaces of interest. We will present a several upper bounds as well as lower bounds of this kind. It turns out the both bounds can be used in the damping optimization process.
The upper bounds of the norm of the matrix of sines of canonical angles determine the quality of the approximations and can be used to control the error bound for the splitting the corresponding GEP in two (or more) independent parts (usually this procedure is called decoupling).
On the other hand the lower bounds can be used to measure the influence of the external damping on the part of the spectrum which is of interest. This bound can be used if one wants to assure that the damping will influence on the one particular part of the spectrum, or if one wants to remove some eigenvalues from the certain region.