The numerics of matrix valued rational approximations

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Vector Fitting (VF) is a powerful and widely used method of constructing rational approximants that provides a least squares fit to frequency response measurements. The original method is based on a least-squares fit to the measurements by a rational function, using an iterative reallocation of the poles of the approximant. We show that one can improve the performance of VF significantly, by using a particular choice of frequency sampling points and properly weighting their contribution based on quadrature rules that connect the least squares objective with an $H_2$ error measure. In this way, the original transfer function is approximated with better global fidelity (as measured with respect to the $H_2$ norm), than the localized least squares approximation implicit in VF. This framework can be extended also to incorporate derivative information, leading to rational approximants that minimize system error with respect to a discrete Sobolev norm. One important issue that arises for VF on matrix-valued functions (transfer functions associated with finite-dimensional multi-input/multi-output (MIMO) dynamical systems), that has remained largely unaddressed, is the control of the McMillan degree of the resulting rational approximant; the McMillan degree can grow very high in the case of large input/output dimensions. We discuss two mechanisms for controlling the McMillan degree of the final approximant, one based on alternating least-squares minimization and one based on ancillary system-theoretic model order reduction methods, such as balanced truncation and realization independent IRKA. Further, we discuss subtle numerical details related to notoriously ill-conditioned Cauchy-type matrices that naturally appear in the barycentric form of the rational approximants.