The numerics of matrix valued rational approximations

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Vector Fitting (VF) is a powerful and widely used method of constructing rational approximants that provides a least squares fit to frequency response measurements. The original method is based on a least-squares fit to the measurements by a rational function, using an iterative reallocation of the poles of the approximant. We show that one can improve the performance of VF significantly, by using a particular choice of frequency sampling points and properly weighting their contribution based on quadrature rules that connect the least squares objective with an H_2 error measure. In this way, the original transfer function is approximated with better global fidelity (as measured with respect to the H_2 norm), than the localized least squares approximation implicit in VF. This framework can be extended also to incorporate derivative information, leading to rational approximants that minimize system error with respect to a discrete Sobolev norm. One important issue that arises for VF on matrix-valued functions (transfer functions associated with finite-dimensional multi-input/multi-output (MIMO) dynamical systems), that has remained largely unaddressed, is the control of the McMillan degree of the resulting rational approximant; the McMillan degree can grow very high in the case of large input/output dimensions. We discuss two mechanisms for controlling the McMillan degree of the final approximant, one based on alternating least-squares minimization and one based on ancillary system-theoretic model order reduction methods, such as balanced truncation and realization independent IRKA. Further, we discuss subtle numerical details related to notoriously ill-conditioned Cauchy-type matrices that naturally appear in the barycentric form of the rational approximants.